

Scalar hairy black holes and solitons in asymptotically flat spacetimes

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A numerical analysis shows that a class of scalar-tensor theories of gravity with a scalar field minimally and nonminimally coupled to the curvature allows static and spherically symmetric black hole solutions with scalar-field hair in asymptotically flat spacetimes. In the limit when the horizon radius of the black hole tends to zero, regular scalar solitons are found. The asymptotically flat solutions are obtained provided that the scalar potential $V(\phi)$ of the theory is not positive semidefinite and such that its local minimum is also a zero of the potential, the scalar field settling asymptotically at that minimum. The configurations for the minimal coupling case, although unstable under spherically symmetric linear perturbations, are regular and thus can serve as counterexamples to the no-scalar-hair conjecture. For the nonminimal coupling case, the stability will be analyzed in a forthcoming paper.

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It was more than thirty years ago when Ruffini and Wheeler [1] proposed the so called no-hair conjecture for black holes. This conjecture states that black holes are completely characterized by their mass, charge, and angular momentum. In order to prove this conjecture, some people established several no-hair theorems in theories which couple classical fields to Einstein gravity, notably the black hole uniqueness theorems in Einstein-Maxwell (EM) theory [2] which establishes that all the solutions of black holes in EM theory are stationary and axially symmetric and contained within the Kerr-Newman family. Other theorems proved by Chase [3], Bekenstein [4], Hartle [5] and Teitelboim [6], showed that stationary black holes solutions are hairless in a variety of theories coupling different classical fields to Einstein gravity. A key ingredient in the no-hair-theorem proofs relies on the assumption of asymptotic flatness (AF) and on the nature of the energy-momentum tensor. For instance, in recent years counterexamples to the no-hair conjecture were found in several theories with non-Abelian gauge fields which include the Einstein-Yang-Mills [7], Einstein-Yang-Mills-Higgs (EYMH) [8], Einstein-Yang-Mills dilaton (EYMD) [9], the Einstein-Skyrme [10] and the Einstein-non-Abelian-Proca [8] theories. The existence of solitons and hairy black holes in these theories [11] is associated with the non-Abelian gauge fields present in them [12]. In the case of matter composed by a single scalar field minimally coupled (MC) to gravity, Sudarsky [13], has proved a very simple no-hair theorem. Among other assumptions, the validity of that theorem relies on the fulfillment of the weak energy condition (WEC) which constrains the scalar potential to be positive semi-definite. Recently a more general no-hair theorem for black holes has been proved. This rules out a multicomponent scalar field coupled minimally to gravity satisfying the WEC, but its field Lagrangian is not quadratic in the field derivatives [14]. In fact a central argument for probing all these theorems relies on the fulfillment of the WEC which is generally believed to be satisfied for all physically reasonable classical matter [15].

In the case of nonminimally coupled (NMC) scalar fields, there are only a few no-hair theorems [16, 17]. The AF hairy black hole solution corresponding to a scalar field conformally coupled to gravity (the Bronnikov-Melnikov-Bocharova-Beckenstein solution [18]) which is often used as a counterexample for the failure of the no-scalar-hair conjecture, has been criticized as a non genuine black hole solution in that the Einstein field equations are not verified at the horizon [19], and moreover, in that it is not a solution at all if one demands a bounded scalar field throughout the static region [12(a)].

Recently, in the context of asymptotically AdS spacetimes, scalar-hairy black holes (SHBH) were found for some scalar-field potentials $V(\phi)$ [20]. A further analysis along this line [21], suggested in addition that by adjusting the parameters of $V(\phi)$, the effective cosmological constant could cancel out so that asymptotically flat SHBH could exist.

Encouraged by the findings in the AdS context, we show in this letter numerical evidence of SHBH solutions in

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static, spherically symmetric and asymptotically flat spacetimes. The solutions are regular throughout the static region (from the Killing horizon to spatial infinity). In particular, for the MC case, the main assumption for the Sudarsky's no-hair theorem [13] not to be valid here is that the class of scalar-field potentials $V(\phi)$ used to construct the solutions are not positive-semidefinite (the WEC is violated). Moreover, the class $V(\phi)$ is in fact such that $V(\phi_c) = 0 = \partial_\phi V|_{\phi_c}$ and $\partial_\phi^2 V|_{\phi_c} > 0$, that is, a root of $V(\phi)$ and a local minimum are located at the same place. This feature and the fact that the non-trivial scalar field settles asymptotically at the local minimum (and therefore also at a root) of $V(\phi)$, leads to configurations that truly represent asymptotically flat solutions. When taking the limit $r_h \rightarrow 0$ (r_h being the horizon radius), the SHBH tend to regular scalar solitons (scalarons). Such results can extend to the NMC case as it turns numerically (the no-hair theorems for the NMC are avoided by considering the scalar potentials as mentioned above).

The SHBH solutions although genuine asymptotically flat solutions, and perhaps the first examples with a regular horizon and with an explicit scalar field potential, constitute only a “weak” counterexample to the no-scalar-field-hair conjecture since they turn to be unstable with respect to radial perturbations.

Einstein-scalar field equations and Lagrangian.— We will consider a model of a scalar field NMC to gravity and with a potential. The simplest model of this kind is obtained by considering the Lagrangian

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{16\pi} R + \xi \phi^2 R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right] . \quad (1)$$

By choosing the NMC constant $\xi = 0$, the above theory corresponds to the minimal coupling case, and $\xi = -1/12$ corresponds to the conformally coupled scalar field (units where $G_0 = c = 1$ are employed). The gravitational field and scalar field equations following from the Lagrangian (1) can be written as

$$G_{\mu\nu} = 8\pi T_{\mu\nu} , \quad \square \phi + 2\xi R = \frac{\partial V(\phi)}{\partial \phi} , \quad (2)$$

where

$$T_{\mu\nu} = G_{\text{eff}} \left\{ (\nabla_\mu \phi) \nabla_\nu \phi - g_{\mu\nu} \left[\frac{1}{2} (\nabla \phi)^2 + V(\phi) \right] + 4\xi [\nabla_\mu (\phi \nabla_\nu \phi) - g_{\mu\nu} \nabla_\lambda (\phi \nabla^\lambda \phi)] \right\} , \quad (3)$$

is an effective energy-momentum tensor which includes all the contributions of the scalar field, and G_{eff} is an effective gravitational “constant” which explicitly depends on the scalar field:

$$G_{\text{eff}} = \frac{1}{(1 + 16\pi\xi\phi^2)} . \quad (4)$$

We will focus on a metric describing spherical and static spacetimes:

$$ds^2 = -Ne^{2\delta} dt^2 + N^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) , \quad (5)$$

where $N \equiv (1 - 2m(r)/r)$ and the functions m and δ depend only on the coordinate r . For the scalar field we also assume $\phi = \phi(r)$. The resulting field equations are

$$\partial_r m = 4\pi r^2 E , \quad (6)$$

$$\partial_r \delta = \frac{4\pi r}{N} [E + S^r] , \quad (7)$$

$$\partial_{rr}^2 \phi = - \left[\frac{2}{r} + \frac{2}{N} \left\{ 2\pi r (S^r - E) + \frac{m}{r^2} \right\} \right] \partial_r \phi + \frac{1}{N} \left[\frac{\partial V(\phi)}{\partial \phi} - 2\xi \phi R \right] , \quad (8)$$

where $E = -T_t^t$, $S^r = T_r^r$ and R are the local energy density, the radial pressure and the Ricci scalar, respectively, given by the following expressions containing no-second order derivatives:

$$E = \frac{G_{\text{eff}}}{1 + 16\pi\xi r \phi (\partial_r \phi) G_{\text{eff}}} \left[\frac{-4\xi \phi (\partial_r \phi) m}{r^2} - 16\pi\xi r \phi (\partial_r \phi) G_{\text{eff}} \left(\frac{N(\partial_r \phi)^2}{2} - V(\phi) - \frac{8\xi \phi (\partial_r \phi) N}{r} \right) \right] + \frac{G_{\text{eff}}}{1 + 192\pi\xi^2 \phi^2 G_{\text{eff}}} \left[\frac{N(\partial_r \phi)^2}{2} (1 + 8\xi + 64\pi\xi^2 \phi^2 G_{\text{eff}}) + 4\xi \phi \frac{\partial V(\phi)}{\partial \phi} + V(\phi) (1 - 64\pi\xi^2 \phi^2 G_{\text{eff}}) \right] , \quad (9)$$

$$S^r = \frac{G_{\text{eff}}}{1 + 16\pi\xi r \phi (\partial_r \phi) G_{\text{eff}}} \left[\frac{-4\xi \phi (\partial_r \phi) m}{r^2} + \frac{N(\partial_r \phi)^2}{2} - V(\phi) - \frac{8\xi \phi (\partial_r \phi) N}{r} \right] , \quad (10)$$

$$R = \frac{8\pi G_{\text{eff}}}{1 + 192\pi\xi^2 \phi^2 G_{\text{eff}}} \left[N(\partial_r \phi)^2 (1 + 12\xi) + 4V(\phi) + 12\xi \phi \frac{\partial V(\phi)}{\partial \phi} \right] . \quad (11)$$

Boundary conditions and numerical methodology.- For black hole configurations we demand regularity on the event horizon r_h and for scalarons regularity at the origin $r = 0$. This implies the following conditions for the fields (hereafter, the subscripts ‘ h ’ stand for a quantity to be evaluated at r_h):

$$m_h = \frac{r_h}{2} , \quad \delta(r_h) = \delta_h , \quad \phi(r_h) = \phi_h , \quad (\partial_r \phi)_h = \frac{r_h [(\partial_\phi V)_h - 2\xi \phi_h R_h]}{4\pi r_h^2 (S_h^r - E_h) + 1} , \quad (12)$$

where

$$R_h = \frac{32\pi G_{\text{eff}}^h}{1 + 192\pi \xi^2 \phi_h^2 G_{\text{eff}}^h} [V_h + 3\xi \phi_h (\partial_\phi V)_h] , \quad (13)$$

$$S_h^r - E_h = -\frac{G_{\text{eff}}^h}{1 + 192\pi \xi^2 \phi_h^2 G_{\text{eff}}^h} [4\xi \phi_h (\partial_\phi V)_h + V_h (1 - 64\pi \xi^2 \phi_h^2 G_{\text{eff}}^h)] - G_{\text{eff}}^h V_h . \quad (14)$$

The value δ_h is fixed so that the desired asymptotic behavior (see below) is obtained. For the scalaron case, regularity at the origin $r = 0$ results by taking the limit $r_h \rightarrow 0$ in the above regularity conditions. In addition to the regularity conditions, we impose asymptotically flat conditions on the space-time (for black holes and scalarons). These imply the following conditions on fields when $r \rightarrow \infty$,

$$m(\infty) = M_{\text{ADM}} , \quad \delta(\infty) = 0 , \quad \phi(\infty) = \phi_\infty . \quad (15)$$

Here M_{ADM} , is the ADM-mass associated with a given configuration. Actually, the value ϕ_∞ will correspond to the local minimum (which is also a root) of $V(\phi)$ (see below).

For a given scalar field potential and for a fixed ξ (i.e., for a given theory), the family of SHBH configurations will be parametrized by the arbitrary free parameter r_h which specifies the location of the black hole horizon. As explained below, ϕ_h is a shooting parameter rather than an arbitrary boundary value, and its value is determined so that the above AF conditions are fulfilled. Therefore for SHBH, $M_{\text{ADM}} = M_{\text{ADM}}(r_h)$. As mentioned above, the scalarons are contained as a limiting case when $r_h \rightarrow 0$ (in this limit, ϕ_0 , the value of ϕ at $r = 0$, is the shooting parameter), and the corresponding configuration is characterized by a unique M_{ADM} .

Using the above system of field equations together with the regularity and asymptotic conditions, we have performed a numerical analysis for one class of scalar field potential and for different values of ξ .

Numerical results. We choose the following asymmetric scalar-field potential leading to the desired asymptotically flat solutions:

$$V(\phi) = \frac{\lambda}{4} \left[(\phi - a)^2 - \frac{4(\eta_1 + \eta_2)}{3} (\phi - a) + 2\eta_1 \eta_2 \right] (\phi - a)^2 , \quad (16)$$

where λ , η_i and a are constants. For this class of potentials one can see that, for $\eta_1 > 2\eta_2 > 0$, $\phi = a$ corresponds to a local minimum, $\phi = a + \eta_1$ is the global minimum and $\phi = a + \eta_2$ is a local maximum. The key point in the shape of the potential, $V(\phi)$, for the asymptotically flat solutions to exist, is that the local minimum $V_{\text{min}}^{\text{loc}} = V(a)$ is also a zero of $V(\phi)$ [22]. The dynamics of scalar fields with such a potential has been analyzed in the past within the aim of studying quantum tunneling from the false vacuum through the true vacuum (see Ref. [23]). A particle-mechanics analogy [23], helps to understand the existence of classical solutions that interpolate between the local minimum $\phi = a$ and a value, $\phi = \phi_{h,0}$, near the global minimum, and rolling across the local maximum. This requires a suitable shooting method in order to find the correct value $\phi_{h,0}$ for the scalar field to reach the local minimum asymptotically [24]. We have enforced such a method here. The asymptotic behavior of the scalar field shows that $\phi(r) \sim a + \text{const.} e^{\pm \sqrt{\lambda \eta_1 \eta_2} r} / r$. The shooting method allows to eliminate the runaway solutions.

Since $V(\phi)$ is not positive definite (we assume $\lambda > 0$), the WEC is violated. On the other hand, taking potentials with extrema different from zero ($V_{\text{min}}^{\text{loc}} \neq 0$), leads to solutions that are asymptotically AdS (if $V_{\text{min}}^{\text{loc}} < 0$), or asymptotically de Sitter (if $V_{\text{min}}^{\text{loc}} > 0$ and provided that suitable boundary conditions are imposed at the cosmological event horizon) with $V_{\text{min}}^{\text{loc}}$ acting as an effective cosmological constant Λ_{eff} (cf. Ref. [20] for an analysis with $\Lambda_{\text{eff}} \neq 0$).

For the numerical analysis, λ determines the scale of different quantities. In particular $r_c = 1/\sqrt{\lambda}$ has been used as a length scale.

Case $\xi = 0$. Figures 1 and 2, show examples of numerical asymptotically flat SHBH and scalaron solutions respectively, for the potential $V(\phi)$. As depicted by those figures, a generic behavior of the solution for ϕ is that it decays exponentially ($\phi \sim \text{const.} e^{-\sqrt{\lambda \eta_1 \eta_2} r} / r$) to the local minimum $V_{\text{min}}^{\text{loc}}$ before reaching the asymptotic value at $\phi = a$ ($a = 0$ in these examples). $\phi_{h,0}$ is the shooting value for which ϕ rolls up to $\phi = a$ asymptotically. The right

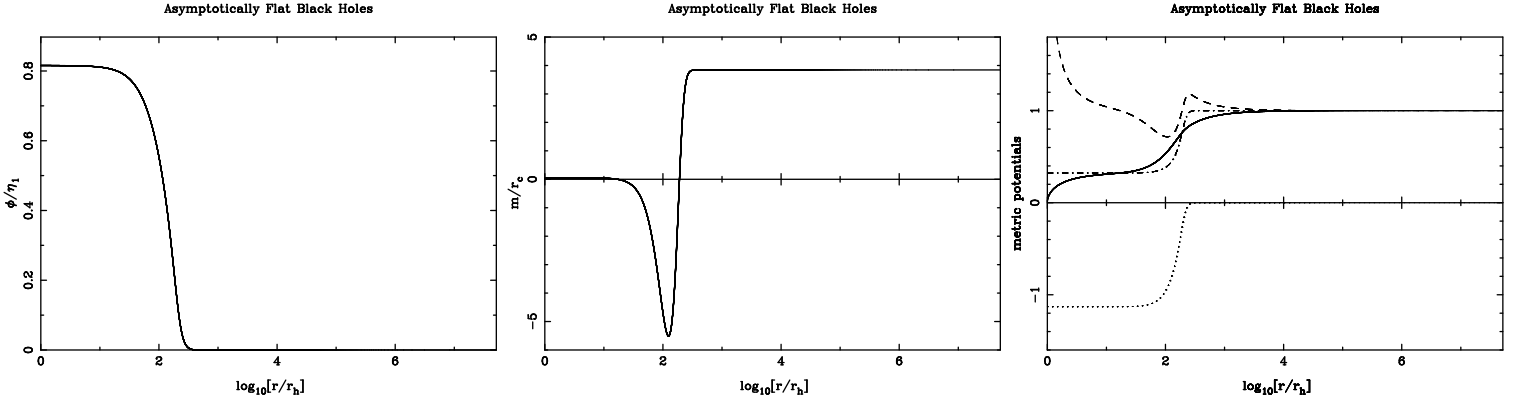


FIG. 1:

Black hole configuration constructed with $\xi = 0$ and $V(\phi)$ as given by Eq. (16) with parameters $\eta_1 = 0.5$, $\eta_2 = 0.1$, $a = 0$, and $r_h = 0.1/\sqrt{\lambda}$, $\phi_h \sim 0.40786$. The left and middle panels depict the scalar field and the mass function respectively. The latter converges to $M_{\text{ADM}} \sim 3.843/\sqrt{\lambda}$. The right panel depicts the metric potentials $\sqrt{-g_{tt}}$ (solid line), $\sqrt{g_{rr}}$ (dashed line), e^δ (dash-dotted line) and δ (dotted line).

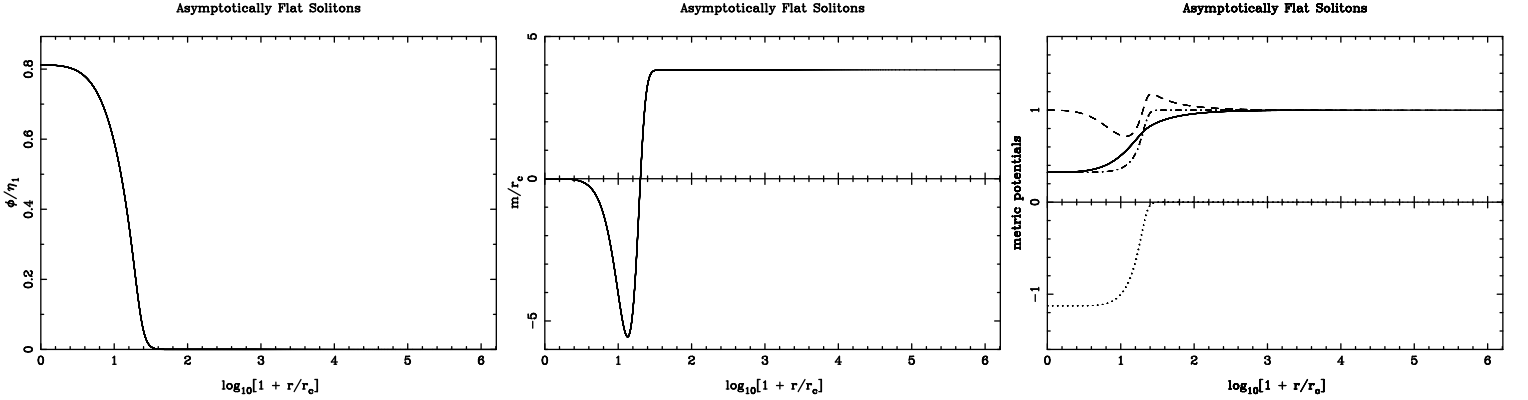


FIG. 2:

Same as Fig.1 for the soliton case. Here $\phi_0 \sim 0.40594$ and $r_c = 1/\sqrt{\lambda}$. Note that the solutions are globally regular, notably at the origin, and $M_{\text{ADM}} \sim 3.827/\sqrt{\lambda}$.

panel of fig.1 (fig.2) shows that the SHBH (scalaron) solution asymptotically approximates the Schwarzschild BH (Schwarzschild “exterior”) solution with mass $M_{\text{ADM}}^{\phi \neq 0}$ respectively.

The ADM mass, $M_{\text{ADM}}(r_h)$, for different values of the parameters $(\lambda, \eta_1, \eta_2)$, turns to be larger than the corresponding mass of the Schwarzschild BH while keeping fixed r_h . It seems therefore that the lowest bound for M_{ADM} is $M_{\text{ADM}}^{\phi=0}$ which corresponds to the hairless Schwarzschild BH. In the limit $r_h \rightarrow 0$, the lowest bound is then $M_{\text{ADM}} = 0$ which is no other but the trivial soliton of the Minkowski spacetime. The theory also admits the no-hairy solutions $\phi(r) = \phi_{\min}^{\text{global}}$ ($\phi(r) = \phi_{\max}^{\text{local}}$), for which ϕ settles at the global minimum (local maximum) V_{\min}^{global} (V_{\max}^{local}) respectively. However, those solutions are not asymptotically flat, but rather correspond to the Schwarzschild AdS solution (Schwarzschild dS solution) with V_{\min}^{global} (V_{\max}^{local}) playing the role of a negative (positive) cosmological constant.

Since $M_{\text{ADM}}^{\phi \neq 0}(r_h) > M_{\text{ADM}}^{\phi=0}(r_h)$, i.e., $M_{\text{ADM}}(r_h)$ is bounded from below (while keeping fixed r_h which in these coordinates is equivalent to fixing the area, A_h , of the horizon) with $M_{\text{ADM}}^{\phi=0}$ corresponding to the Schwarzschild black hole, heuristically one expects that the hairy BH configurations found here with fixed AF boundary conditions are unstable, since the stable ones would correspond to those with the lowest $M_{\text{ADM}}(r_h)$ (cf. Ref. [25]). Such a behavior is a signature of the unstable nature of the SHBH configurations which is confirmed by a perturbation analysis (similar arguments can be applied to the scalarons to exhibit their unstable nature).

Case $\xi \neq 0$. After performing numerous experiments for the minimal coupling case $\xi = 0$, we also found (for some set of values $\xi > 0$) that asymptotically flat hairy black holes and scalarons exist for the NMC case as well (the

resulting figures are qualitatively similar to Figs.1 and 2). However, a more exhaustive analysis is to be performed in this case. For instance, the stability in the NMC is an issue that remains to be investigated. This will be the object of a forthcoming paper in which we also plan to provide a full sample of black hole and soliton configurations, and to analyze more carefully the quantitative behavior of $M_{\text{ADM}} = M_{\text{ADM}}(r_h, \xi)$ for a larger range of ξ [27].

Linear stability analysis.- We now turn to the question of the stability of the asymptotically flat space-times presented here. For simplicity, the following analysis is limited to the case $\xi = 0$. We consider only spherically symmetric time-dependent linear perturbations to the spherically symmetric static metric (5) and the scalar field configuration discussed in the previous section. These perturbations will be parametrized by

$$\tilde{\phi}(t, r) = \phi(r) + \phi_1(t, r) , \quad (17)$$

$$\tilde{g}_{tt}(t, r) = g_{tt}(r)(1 - h_0(t, r)) , \quad (18)$$

$$\tilde{g}_{rr}(t, r) = g_{rr}(r)(1 + h_2(t, r)) , \quad (19)$$

where $g_{tt}(r) = -Ne^{2\delta}$, $g_{rr}(r) = N^{-1}$, $\phi(r)$ represent the non-perturbed solution and $\phi_1(t, r)$, $h_0(t, r)$, $h_2(t, r)$ denote a small perturbation to the non-perturbed solution. It is straightforward to obtain the metric perturbations as a function of the scalar field perturbation

$$h_2 = 8\pi r(\partial_r \phi)\phi_1 , \quad \partial_r h_0 = \partial_r h_2 - 16\pi r(\partial_r \phi)(\partial_r \phi_1) , \quad (20)$$

and the linear Schrödinger type equation for the scalar field perturbation

$$-\partial_{r_* r_*}^2 \psi + V_{\text{eff}}^*[r_*]\psi = -\partial_{tt}^2 \psi , \quad \frac{dr_*}{dr} = \frac{e^{-\delta}}{N} , \quad (21)$$

where we have used $\psi \equiv r\phi_1$, $r_*(r)$ is the “tortoise” coordinate, and the effective potential is given by

$$V_{\text{eff}}(r) = Ne^{2\delta} \left[\frac{N}{r} \left\{ \partial_r \delta + \frac{\partial_r N}{N} \right\} - 8\pi r N (\partial_r \phi)^2 \left\{ \partial_r \delta + \frac{\partial_r N}{N} + \frac{1}{r} \right\} + 16\pi r (\partial_r \phi) \frac{\partial V}{\partial \phi} + \frac{\partial^2 V}{\partial \phi^2} \right] . \quad (22)$$

Figure 3 depicts V_{eff} for the SHBH (left panel) and soliton (right panel) respectively.

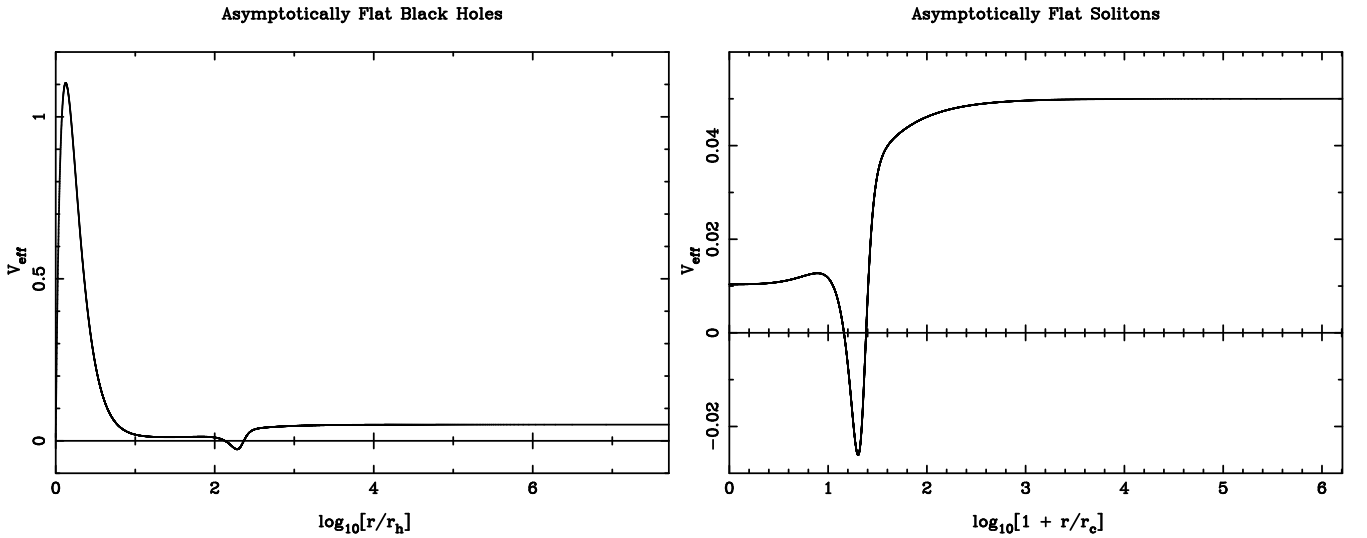


FIG. 3:

The potential V_{eff} corresponding to the SHBH solution of fig.1 (left panel) and to the soliton solution of fig.2 (right panel).

One can seek mode perturbations $\psi(r) = \chi(r)e^{i\sigma t}$, so that the Equation (21) writes $(-D^a D_a + V_{\text{eff}})\chi = \sigma^2 \chi$ where D_a is the derivative operator associated with an auxiliary metric of a manifold M . According to the theorem proved in [26], a sufficient condition for the static and spherically symmetric configurations to be unstable, is that the operator $A = -D^a D_a + V_{\text{eff}}$ be negative in the Hilbert space $L^2(M)$. This theorem can be easily implemented when the background (static) solution is given analytically. For instance, one can see explicitly, that the Schwarzschild

solution $\delta(r) = 0$, $N = 1 - 2M_{\text{ADM}}^{\phi=0}/r$ is stable within these kind of perturbations and within this class of potentials, since $V_{\text{eff}}(r) = N \left[\frac{2M_{\text{ADM}}^{\phi=0}}{r^3} + \lambda\eta_1\eta_2 \right] > 0$ in the region $r_h \leq r \leq \infty$. A result, which is heuristically confirmed by the fact that $M_{\text{ADM}}^{\phi=0}$ is a lower bound within the set of configurations with fixed r_h [27]. However, when the background solution is given numerically the implementation of the theorem is less trivial since it requires the use of a suitable square integrable vector Ψ in $L^2(M)$ to compute $\langle \Psi, A\Psi \rangle$. Instead, we have opted to explicitly solve the associated Schrödinger-like equation of Eq. (21) with respect to the original r coordinate to find a “bound state” with negative σ^2 . Imposing regularity at r_h (origin), i.e., $\chi|_{r_h,0} = 0$, and demanding that $\chi(r) \rightarrow 0$ for $r \rightarrow \infty$ we found $\sigma^2 \sim -0.00241$ and $\sigma^2 \sim -0.00243$ for the SHBH and scalarons solutions respectively (see Fig. 4). This shows that the modes $\psi(r) = \chi(r)e^{i\sigma t}$ can grow unboundedly with time, and therefore leading to the conclusion that the SHBH and soliton configurations are unstable.

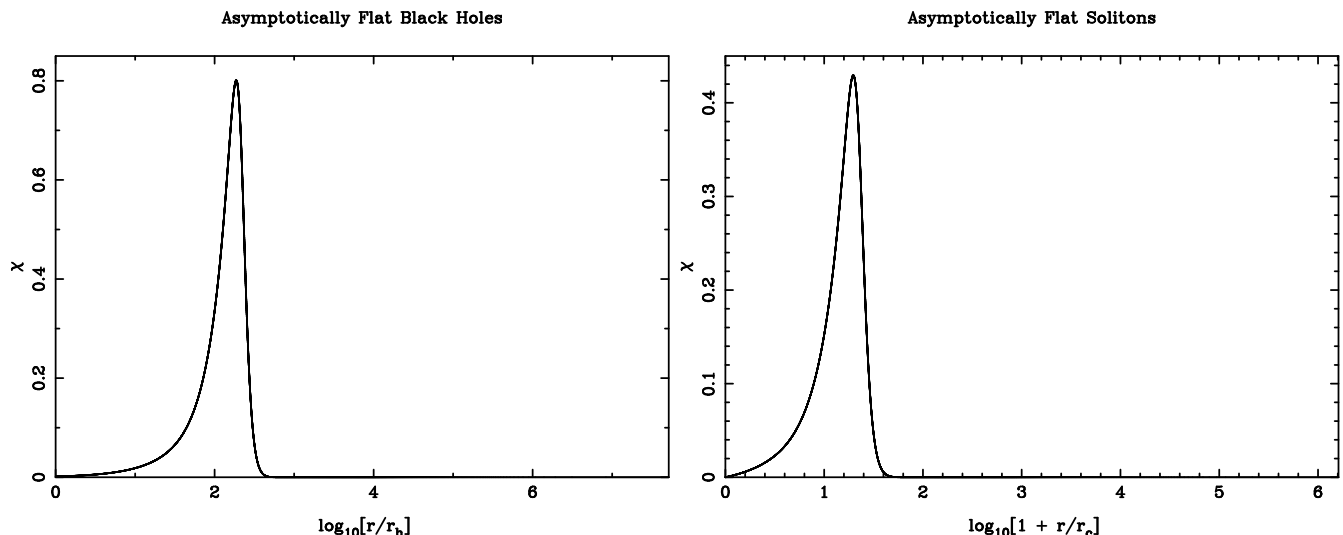


FIG. 4:

The mode χ corresponding to the perturbation of the SHBH solution of fig.1 (left panel) with $\sigma^2 \sim -0.00241$ and the soliton solution of fig.2 (right panel) with $\sigma^2 \sim -0.00243$ respectively.

Conclusions. We have found black hole solutions that support a nontrivial scalar hair in spherical, static and asymptotically flat spacetimes. Such solutions are perhaps the first regular ones throughout the static region (from the horizon to spatial infinity) with an explicit scalar field potential [28]. In the limit where $r_h \rightarrow 0$, regular scalar solitons (scalarons) are obtained. In the minimal coupling case $\xi = 0$, the solutions turn to be unstable with respect to linear-spherical perturbations and their ADM mass turns to be larger than the corresponding Schwarzschild black hole while keeping fixed r_h . Therefore, these are to be regarded as weak counterexamples to the no-scalar-hair conjecture. For nonminimal couplings $\xi \neq 0$, similar solutions are found, however their stability remains to be investigated. Another analysis to be performed are the collapse of these hairy black holes violating WEC and the possibility of having hairy black holes with nodes in the scalar field using higher order potentials. An interesting outcome of the above results is that it opens the possibility for a further study of the solutions in the context of the isolated horizon formalism (cf. Ref. [29]).

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